Suggested Errata for Cont, Tankov (2004) "Financial Modelling with Jump Processes"

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This document lists some potential typos/inconsistencies in the notation of the 2004 printing that are neither included in the errata published under http://www.cmap.polytechnique.fr/~rama/Jumps/ nor in an updated PDF version of some of the book chapters.

Cover

• There is a typo in the third paragraph of the back cover - replace "similes" by "smiles".

Chapter 2 - Basic Tools

Page 47

• In the proof to Proposition 2.11, the integrand in the first line should should be $\lambda^{n+1}e^{-\lambda t_{n+1}}$ instead of $\lambda^n e^{-\lambda t_{n+1}}$ since this is the density of $(T_1, T_2, \ldots, T_{n+1})$. It is correct in the second line where dt_{n+1} has been integrated out.

Page 51

• Equation (2.75) should read

$$N_t' = \sum_{n \ge 1} \mathbf{1}_{t > T_n},$$

i.e the lower index of summation should be one.

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• Equation (2.76) should read

$$X_t = \sum_{n \ge 1} \mathbf{1}_{t \ge T'_n},$$

i.e the inequality sign is the wrong way around.

Page 56

• Equation (2.85) should read

$$\frac{d}{dt}N_t(\omega) = M(\omega, dt).$$

Page 57

• In Definition 2.18, there is one μ too much. The first sentence should read "...and μ a given (positive) Radon measure on (E, \mathcal{E}) .".

Chapter 3 - Lévy Processes

Page 71

• There is an "a" missing in the first sentence of Proposition 3.3. It should start with $(X_t)_{t\geq 0}$ is a compound Poisson process...".

Page 74

• The conditioning in the first equality of the proof to Proposition 3.4 is the wrong way around instead of

$$E\left[E\left[\exp \mathrm{i} u.X_t\right]\right|N_t\right]$$

it should read

$$E\left[E\left[\exp \mathrm{i} u.X_t | N_t\right]\right].$$

Page 82

• In the first sentence it should either be "...exponential formula in (3.9)..." or "...exponential formula in Proposition 3.6...".

• I think the definition of X_t^c in the second last paragraph should be $X_t^c = X_t - X_t^l - \lim \tilde{X}_t^{\epsilon}$ so that X_t^c is the continuous part of X. This is also in accordance with the way it is used in the proof of Theorem 3.1 on page 84.

Page 82

• To be consistent with the definition of \tilde{X}_t^{ϵ} on page 80 and γ_c later on the same page, I think the indicator in the characteristic exponent should be $1_{|x|<1}$ instead of $1_{|x|\leq 1}$. This applies three times on this page and a few more times in what follows.

Page 86

• In the proof of Proposition 3.9, the second equation should read

where
$$\tilde{X}_t^{\epsilon} = \int_{\epsilon \le |x| < 1, s \in [0,t]} x \tilde{J}_X(ds \times dx)$$

since this is how \tilde{X}_t^{ϵ} was defined on page 80. The compensation term is then of finite variation by Equation (3.16).

Page 93

• We have

$$\left(\frac{B_{at}}{\sqrt{a}}\right)_{t\geq 0} = \left(\frac{W_{at}}{\sqrt{a}} + \sqrt{a\gamma t}\right)_{t\geq 0} = \left(W_t + \sqrt{a\gamma t}\right)_{t\geq 0} \neq \left(B_t + \sqrt{a\gamma t}\right)_{t\geq 0}.$$

• In the third paragraph, the characteristic function should be defined as $\Phi_{X_t}(z) = \exp[t\phi(z)]$ instead of $\Phi_{X_t}(z) = \exp[-t\phi(z)]$.

Chapter 4 - Building Lévy Processes

Page 105

• In the first line, there is a "the" or "a" missing in front of "call option price".

Page 111

• Equation (4.13) is the jump size distribution not the Lévy measure. So either change $\nu(dx)$ to f(dx) or multiply the whole equation by λ .

• I think the characteristic exponent of the Kou model should be

$$\Psi(u) = -\frac{\sigma^2 u^2}{2} + ibu + iu\lambda \left\{ \frac{p\lambda_+}{\lambda_+ - iu} - \frac{(1-p)\lambda_-}{\lambda_- + iu} \right\},$$

i.e. the numerator of the two fractions is incorrect.

• The cumulants and moments of the Kou model seem to be wrong. I get

$$\operatorname{Var} X_{t} = t \left(\sigma^{2} + 2\lambda \left(p/\lambda_{+}^{2} + (1-p)/\lambda_{-}^{2} \right) \right),$$

$$c_{3} = 6t\lambda \left(p/\lambda_{+}^{3} - (1-p)/\lambda_{-}^{3} \right),$$

$$c_{4} = 24t\lambda \left(p/\lambda_{+}^{4} + (1-p)/\lambda_{-}^{4} \right).$$

Chapter 7 - Modelling Financial Time Series with Lévy Processes

Page 207

• In the second paragraph it should either read "...for using continuous time models..." or "...for using a continuous time model...".

Page 220

• In Equation (7.14), the "hat" is misplaced in the denominator - it should be $\hat{\sigma}(\Delta)^4$ instead of $\hat{\sigma}(\Delta)^4$.

Page 221

• Equation (7.15) looks strange. If Value-at-Risk corresponds to the maximum daily loss, then I'd have expected

$$P(W_0r_t(\Delta) \leq \operatorname{VaR}(p, t, \Delta)) = p.$$

Page 223

The indices seems to be a little inconsistent. First it says "...time series of n..." but then there are n+1 indices, namely from 0 to nΔ. In Equations (7.18) and (7.19) there are only n indices as the k = 0 is omitted and the first return is not t = 0 any more but a general t + Δ.

- In the third last paragraph, $\Phi_t(z) = \exp[-t\psi(z)]$ should read $\Phi_t(z) = \exp[t\psi(z)]$ (compare to Equation (3.1) on page 70).
- In the last sentence of the third last paragraph, replace "...relation for ϕ :" by "...relation for ψ :".

Page 232

This comment is very similar to the one for page 223. You write "sample of N returns" but then introduce N + 1 indices. Consequently, Equations (7.41) and (7.42) are the realized and sample variance on [0, T + Δ]. You might want to change t = 0, Δ, ..., NΔ to t = 0, Δ, ... (N - 1)Δ. Also the indices in Equations (7.41) and (7.42) should be r_{t+kΔ} where you sum over k = 1 to N.

Page 234

• In the caption to Figure 7.8, you write "...theoretical value of 32%." But according to the plots, it rather looks like it should be around 20% - 25%.

Page 235

• In the second paragraph, there is a percentage sign missing after 32.5.

Page 236

• The "hat" in the first line is misplaced, it should read $\hat{\sigma}^2$ instead of $\hat{\sigma}^2$.

Chapter 8 - Stochastic Calculus for Jump Processes

Page 249

• In Definition 8.1, there is an "a" missing - it should read "...is a bounded random variable...".

Page 250

• In Example 8.1, there is a "-"-sign at the beginning of the second sentence that probably shouldn't be there.

Page 257

• In the last equation W_{T_i} ended up in the subscript of $W_{T_{i+1}}$.

- In the multiline equation at the top, in the first two equalities, it should read $W_{T_{i+1}} W_{T_i}$ instead of $W_{T_i} W_{T_{i+1}}$.
- In the third equality of the multiline equation at the top, I think the conditioning should be on \mathcal{F}_{T_i} and not on \mathcal{F}_{T_j} as i > j. This way, we use that $\phi_i \phi_j \left(W_{T_{j+1}} W_{T_j} \right)$ is \mathcal{F}_j -measurable and the conditional expectation of $W_{T_{i+1}} W_{T_i}$ is zero.
- Although it doesn't make a difference in the end, there is a "2" missing in front of the second term in the third equality.

Page 260

• In the first paragraph, replace $(A_j)_{i=1...m}$ by $(A_j)_{j=1...m}$.

Page 261

- In the second equation, you suddenly use the notation $\mathbb{E}\left\{\right\}$ instead of $\mathbb{E}\left[\right]$ and then immediately switch back.
- There is some inconsistency in the use of $L^2(\mathbb{P}) / L^2$ (compare e.g. to the last line on page 258 or to the last paragraph on page 276).

Page 263

• In Equation (8.28), the left hand side should read $[X, X]_T$ instead of $[X, X]_t$.

Page 265

- I think the "as" in the first sentence of the 2nd paragraph should be dropped, i.e. "...is decomposed into the sum of...".
- In the first equation, the integrand should have an index different from t, e.g. $\int_0^t a(s) ds$.

Page 266

• There is an "is" missing in Example 8.5. It should read "The quadratic variation of a Lévy process is again a Lévy process...".

Page 272

• I noticed that there is some inconsistency in the placement of the tilde in the compensated jump measure - sometimes it is \tilde{J}_X and sometimes \tilde{J}_X . This applies throughout the book.

- Before the third equation, you write "for any measurable function $f : [0,T] \times \mathbb{R} \to \mathbb{R}$:". But it looks like this only applies to such functions f that are in $C^{1,1}$?
- I think there is a λ missing in the second term of Equation (8.48). Also, the inner integral should be over \mathbb{R} and not over \mathbb{R}^d as J_X is a random measure on $[0, T] \times \mathbb{R}$. It should read

$$\ldots + \int_0^t \lambda ds \int_{\mathbb{R}} F(dy) \left[f\left(s, X_{s-} + y\right) - f\left(s, X_{s-}\right) \right].$$

Page 276

- In Proposition 8.15, the first term on the right hand side should be $f(X_0)$ instead of f(0) for consistency (see e.g. Proposition 8.16).
- For consistency, the lower limit of summation in the first equation in the proof to Proposition 8.15 should be changed to zero as usually $t_0 = 0$ and not $t_1 = 0$.

Page 277

• In the line before Equation (8.54), I think it should read $f \in C^{1,2}([0,T],\mathbb{R})$ instead of $f \in C^{1,2}([0,T] \times \mathbb{R},\mathbb{R})$.

Page 278

• In Equation (8.55), the lower limit of integration should be $-\infty$ instead of ∞ .

Page 279

• In Proposition 8.18, the second term on the left hand side should be $f(0, X_0)$ instead of f(0, 0) for consistency (see e.g. Proposition 8.16).

Page 281

• I think there is a $\frac{1}{2}$ -term missing in front of $f''(X_{s-}) |\Delta X_s|^2$ a few times (e.g. in Equation (8.58) and the last two equations on this page).

Page 284

• In Proposition 8.20, $(X)_{t\geq 0}$ should be replaced by $(X_t)_{t\geq 0}$ for consistency. This applies a few more times throughout the book. See e.g. Proposition 8.21 on the same page.

• In the first equation, replace $e^{\Delta X_t}$ by $e^{-\Delta X_t}$.

Page 287

• I think that the sign of the sum in Equation (8.75) is the wrong way around, it should read

$$X_t = L_t + \frac{\sigma^2 t}{2} - \sum_{0 \le s \le t} \{ 1 + \Delta L_s - e^{\Delta L_s} \}.$$

Chapter 9 - Measure Transformations for Lévy Processes

Page 294

• In the middle of the page, I suppose "...calls and puts: $H = (K - S_T^i)^+$, $H = (K - S_T^i)^+$..." should read "...calls and puts: $H = (K - S_T^i)^+$, $H = (S_T^i - K)^+$...".

Page 295

- In the first bullet point, $1 \ge 1_A \le 0$ should read $1 \ge 1_A \ge 0$.
- Inte the paragraph before Equation (9.6), I suppose $\Pi_0(H) = E^{\mathbb{Q}}[H]$ should read $\Pi_0(H) = e^{-rT} E^{\mathbb{Q}}[H]$.
- In Equation (9.6), we always consider the value $\Pi_0(H)$ at time zero but the discount is done over the interval T t. This should be changed to just T.

Page 296

- In the first sentence of the second paragraph, I suppose $A \to A = e^{rt} \Pi_t(1_A)$ should read $A \to \mathbb{Q}_t(A) = e^{rt} \Pi_t(1_A)$. I am not too sure but the current expression looks wrong to me.
- I don't understand the definition of arbitrage opportunity given at the bottom of the page. As it does not restrict the value process to start at $V_0 = 0$, an investment in the bank account would constitute an arbitrage given this definition. Thus, I think it should be added that $V_0 = 0$. However, even then I do not see why it is not sufficient to have V_0 and there exists a T > 0such that $\mathbb{P}(V_T \ge 0) = 1$ and $\mathbb{P}(V_T > 0) \ne 0$. I.e. why do we require V_t to be non-negative with probability one in [0, T]?

- In the paragraph after Equation (9.9), you write that the payoffs S_T^i and $e^{r(T-t)}S_t^i$ are identical. But this is not true. Under \mathbb{Q} , they are identical in expectation, which is what you write in Equation (9.10).
- In the second line of the same paragraph, replace S_t by S_t^i .
- I think there are some mistakes in the last paragraph. From Ŝ (remove the subscript t for consistency) being a martingale under Q, it does not follow that ∫₀^t φ_udS_u is a martingale under Q. I guess this was supposed to be the discounted portfolio value process (e.g. V̂) and the integral being w.r.t. Ŝ.

Page 299

• In the last paragraph, $\int_0^t \phi dS$ should read $\int_0^t \phi d\hat{S}$.

Page 300

• In Proposition 9.3, there is a subscript t missing for S^d , i.e. it should read $(S_t^0, S_t^1, \ldots, S_t^d)_{t \in [0,T]}$.

Page 301

• Change "...a martingale \hat{S} with..." to "...a martingale X with...".

Page 305

• Maybe I am totally wrong here, but I don't see where the second term the last two equalities of the second last equation on this page come from. I would have assumed that this is using the stationarity of the increments, i.e.

$$E^{\mathbb{P}}\left\{g\left(X_{t}-X_{s}\right)\frac{D_{t}}{D_{s}}\right\}=E^{\mathbb{P}}\left\{g\left(X_{t-s}\right)D_{t-s}\right\}=E^{\mathbb{Q}}\left\{g\left(X_{t-s}\right)\right\}=E^{\mathbb{Q}}\left\{g\left(X_{t}-X_{s}\right)\right\}$$

Page 307

• In the second equation, replace λ^P and λ^Q by $\lambda^{\mathbb{P}}$ and $\lambda^{\mathbb{Q}}$ for consistency.

Page 308

• In Proposition 9.8, replace "...if and only if three following..." by "...if and only if the three following...".

• In the second paragraph, I suppose the characteristic triplet should be $(0, \nu, \gamma)$ instead of (σ^2, ν, γ) since you consider Lévy processes with no Gaussian component.

Page 311

• In the two equations, there is an indicator missing - replace $x_{|x|\leq 1}$ by $x_{1|x|\leq 1}$ in each of them.

Page 316

• I'm not sure if this is an alternative expression but I guess "...class of martingale measures equivalent..." should read "...class of equivalent martingale measures...".

Chapter 10 - Pricing and Hedging in incomplete Markets

Page 316

- In the first paragraph, change "...jump times and jump sizes is unchanged." to "...jump times and jump sizes are unchanged.".
- For consistency with e.g. Equation (10.7), replace $C^{M}(t, S_{t})$ by $C^{M}(t, S)$ in Equation (10.6).
- In Equation (10.7), replace $-\lambda \exp\left(m + \frac{\delta^2}{2}\right)$ in the exponent by $-\lambda \exp\left(m + \frac{\delta^2}{2}\right)\tau$. Make the same change in the equation for S_n below.

Page 325

- In Equation (10.12), you consider $\mathbb{Q} \in M(S)$ while in the following two equations $\mathbb{Q} \in M_a(S)$. I wonder if Equation (10.12) should be changed to $M_a(S)$. Since $M_a(S)$ is the set of martingale measures absolutely continuous w.r.t \mathbb{P} while M(S) is the set of equivalent martingale measures by the definition on page 321, we have $M(S) \subset M_a(S)$ and thus Equation (10.12) might not hold for some measures considered below.
- In Proposition 10.2, replace "...and Lévy measure verifying ν verifies:" by "...and the Lévy measure ν verifies:".
- I think the upper bound of S_0 in the first point. S_0 is the upper bound on the current option value (i.e. at t = 0). But the upper bound on the expected payoff should then be $e^{rT}S_0$. Alternatively (but that wouldn't match the expression above, where you consider expected payoffs under \mathbb{Q}) the range of current prices is given by $[(S_0 - Ke^{-rT}), S_0]$. In the second point, it seems correct as both the upper and lower bounds are in terms of prices at t = 0.

- In Equation (10.23), $M_e(S)$ is undefined. Should this be $M_a(S)$?
- In Equation (10.25), for consistency, replace $M^{a}(S)$ by $M_{a}(S)$.
- In Equations (10.23) and (10.24), shouldn't this be the expectation of \hat{H} instead of H and the left hand side is the value at t = 0?

Page 330

• In the third line, replace $\pi^{\alpha}(H)$ by $\Pi^{\alpha}(H)$.

Page 331

- The wording "...when the (discounted) price is a martingale measure..." seems odd in the second paragraph under 10.4. It should probably read "..when the (discounted) price is a martingale...".
- In Equation (10.26), then infimum should be taken over V_0, ϕ just as in the equation below.

Page 332

• Under Equation (10.29), replace dS with $d\hat{S}$ (twice, once within Proposition 10.4 and once below) and S by \hat{S} (once below Proposition 10.4).

Page 333

- For consistency (see e.g. Proposition 9.2), replace $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{Q})$ by $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{Q})$
- Regarding Equation (10.30). First, the reference should be to Proposition 3.14 (and without brackets). Also, the condition in Equation (10.30) does not guarantee that S is a martingale, as suggested by "...S is a square integrable martingale if and only if..." but merely that S is square integrable. This is clear from what follows though.
- In the equation below Equation (10.30), $b_X t$ should be $ib_X u$.

Page 334

 The first term in Equation (10.31) comes from the isometry applied to the Brownian motion part of Z. But if Z has characteristic triplet (σ², ·, ·), then shouldn't this term be

$$\int_0^T ||\phi_t \hat{S}_t \sigma^2||^2 dt.$$

• Also in Equation (10.31) for consistency with Proposition 10.5, change $\nu(dz)$ to $\nu_Z(dz)$.

- I suppose S_{t-} should be replaced by \hat{S}_{t-} in the third equation (four times).
- I suppose in Equation (10.32), then infimum should be taken over $V_0, \phi \in L^2(S)$.
- In the last equation before Proposition 10.5, replace H by \hat{H} or explicitly incorporate the discounting.

- In Equation (10.35), shouldn't 1/S be replaced by $1/\hat{S} = e^{rt}/S$? See the last equation on page 336. The same applies to Equation (10.39).
- In the paragraph starting with "and its discounted...", replace $C(0, S_0) = E^{\mathbb{Q}}[H(S_T)]$ by $C(0, S_0) = e^{-rT} E^{\mathbb{Q}}[H(S_T)].$

Page 336

- In the second equation, I think there should be a K term in the second to last inequality.
- In the third equation, replace S_{t-} by \hat{S}_{t-} twice in the first line and once in the second line. In the second line, delete the dt term after the first integral. Both of these typos do not appear in Equation (10.38).
- In the last equation, there are either brackets missing around $C(t, S_{t-}(1+z)) C(t, S_{t-})$ or the second of these term should have a positive sign.

Page 337

• In the equation for $R_T(\phi)$, I think there is a σ^2 missing in the first integral. S_{t-}^2 should be replace by \hat{S}_{t-}^2 in the first integral and S_{t-} by \hat{S}_{t-} in the second. Both comments regarding the first integral also apply to the next equation.

Page 338

• The first equation should read (there are multiple changes)

$$R_T(\phi) = E\left[\int_0^T dt |C(t, S_{t-}(1+a)) - C(t, S_{t-}) - a\hat{S}_{t-}\phi_t|^2\right]$$

so that S_{t-} in the denominator in the second, fourth etc. equation has to be replaced by \hat{S}_{t-} .

- In Equation (10.40), replace $\phi_t dS_t$ by $\phi_u dS_u$.
- In Equation (10.41), replace S_{t-a} by \hat{S}_{t-z} .

- In Equation (10.42), replace Sz by $\hat{S}z$.
- For consistency, replace $S_t = S_0 \exp(-\mu t + N_t)$ by $S_t = S_0 \exp(-\mu t + aN_t)$.

- For consistency, replace $\mathbb{Q} \in M^a(S)$ by $\mathbb{Q} \in M_a(S)$ in Equation (10.49).
- See the comment regarding Equation (10.24) on page 329, I think the right hand side of the second equation should be $E^{\mathbb{Q}^*}[\hat{H}]$.

Page 344

- In Proposition 10.7, Equation (10.50), there is an opening bracket missing in the second row.
- In Proposition 10.7, replace $(Z_t)_{t \in [0,T]}$ by $(X_t^*)_{t \in [0,T]}$.
- In Proposition 10.7, I get when manually computing the dynamics under the risk-neutral measure obtained by the Esscher transform of the process

$$R_t = \int_0^t \frac{dS_t}{S_{t-}}$$

that

$$b^* = b + \beta \sigma^2 + \int_{-1}^{1} \nu(dx) \left[e^{\beta(e^x - 1)} - 1 \right] x.$$

As I get the same expression for Equation (10.50), I suppose that this is a type in the book.

Page 345

• In Example 10.2, change $\sum_{j=1}^{N_t} Y_i$ to $\sum_{j=1}^{N_t} Y_j$ and $\sum_{j=1}^{N_t^*} Y_i^*$ to $\sum_{j=1}^{N_t^*} Y_j^*$.

Page 346

• The first sentence in the second paragraph is not complete - it ends with "...the minimal entropy martingale has.".

Page 347

• The time indices are missing in the integral in Equation (10.53). Given that the stochastic integral is later considered to be a \mathbb{Q} martingale, I suppose that dS should be replaced by $d\hat{S}$.

Chapter 11 - Risk-neutral Modelling with exponential Lévy Processes

Page 354

• In the middle of the page, replace "...for characteristic function of Lévy processes..." by either "...for the characteristic function of Lévy processes..." or "...for characteristic functions of Lévy processes...".

Page 355

• For consistency, replace E^Q by $E^{\mathbb{Q}}$ in Equation (11.9).

Page 356

- My understanding is that at the money forward means that the strike price is equal to the forward price, so $S_t = Ke^{-r(T-t)}$ and $S_t < Ke^{-r(T-t)}$ instead of $S_t = Ke^{r(T-t)}$ and $S_t < Ke^{r(T-t)}$. This is consistent with the later use after Equation (11.10).
- Replace "...in all exp-Lévy models call option price depends..." by "..in all exp-Lévy models the call option price depends..." or "..in all exp-Lévy models call option prices depend...".

Page 357

• For consistency, replace $\Sigma_t(t + \tau, mS(t))$ by $\Sigma_t(t + \tau, mS_t)$.

Page 359

• In the first equation, replace $I_0(K/S_t, T-t)$ by $I_0(T-t, K/S_t)$ for consistency.

Page 361

• For the inverse Fourier transform, the integral should be w.r.t ν . I.e. change dx to $d\nu$.

Chapter 13 - Inverse Problems and Model Calibration

Page 432

- The discounting in the equation should be e^{-rT_i} instead of e^{-rT} . This also applies to the Equations in Calibration Problem 1 and 2.
- In the last paragraph, replace "exponential-Lévy of jump-diffusion models" by "exponential-Lévy or jump-diffusion models".

Chapter 15 - Stochastic Volatility Models with Jumps

Page 470

• In the last paragraph, replace "In X is a..." by "If X is a...".

Page 471

• In the last paragraph of Section 15.0, replace "not time series of underlying" by "not the time series of the underlying".

Page 472

- In the paragraph following Equation (15.3), replace "This parameters..." by "This parameter...".
- In the paragraph preceding Equation (15.4), replace "...instantaneous correlation coefficient one can write" by "...instantaneous correlation coefficient. One can write".

Page 473

• In the last paragraph of Section 15.1.0, replace "...which are fatter normal distributions, ..." by "...which are fatter than normal distributions, ...".

Page 476

• In the caption to Figure 15.1, replace "A typical trajectory of CIR process..." by "A typical trajectory of the CIR process...".

Page 487

• After Equation (15.28), replace $l(u) = E\left[e^{uZ_1}\right]$ by $l(u) = \ln E\left[e^{uZ_1}\right]$.

Page 489

• In Equation (15.31), the second term in the last line should read

$$\sigma_0^2\left(iu\beta-\frac{u^2}{2}\right)\varepsilon(\lambda,t),$$

i.e. the leading ``i" should be deleted.

Page 490

• In Equation (15.35), I believe that the sign of the last term in the numerator of the last fraction should be negative instead of positive.

• The reference at the end of the second paragraph seems wrong. It says "Carr et al. [162]" but the index 162 refers to a paper by Geman et al..