Suggested Errata for Musiela, Rutkowski (2005) “Martingale Methods in Financial Modelling”

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Chapter 3 - Continuous-Time Models

Page 126

- Theorem 3.1.1 first defines $c : \mathbb{R}_+ \times (0, T]$ but then defines ϕ as a function of the call price for every $t \in [0, T]$.

Page 128

- The last equation should read

$$V_t^*(\phi) = V_0^*(\phi) + \int_0^t \sigma S_u^* N\left(d_1\left(S_u, T-u\right)\right) dW_u^* = \dots$$

  I.e. replace the first “S_u” by “S_u^*” using Equation (3.10). Similarly, replace “$\zeta_u = \sigma S_u N\left(d_1\left(S_u, T-u\right)\right)$” by “$\zeta_u = \sigma S_u^* N\left(d_1\left(S_u, T-u\right)\right)$”. In the following inequality on page 129, keeping “S_u” instead of “S_u^*” is valid due to the inequality but might be changed for consistency.

Page 130

- In the line before Equation (3.33), replace “$\xi = -W_T/\sqrt{T}$” by “$\xi = -W_T^*/\sqrt{T}$”.

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- In the first line, the definition "$\mu : [0, T^*] \times \mathbb{R} \to \mathbb{R}$" could equivalently be replaced by "$\mu : [0, T^*] \times \mathbb{R}_+ \to \mathbb{R}$" since S is a strictly positive process.

Page 132

- Before Equation (3.44) the put price is defined by $p : \mathbb{R}_+ \times (0, T] \to \mathbb{R}$ but then Corollary 3.1.4 defines it as $p : \mathbb{R}_+ \times [0, T] \to \mathbb{R}$.

Page 141

- The order of the price function in the sentence before Equation (3.56) should be switched for consistency. I.e. "We write $c\,(S_t, \tau, K, r, \sigma)$ and $p\,(S_t, \tau, K, r, \sigma)$ to denote the price of a call and a put option respectively.". Although clear from the notation, "$p\,(S_t, \tau, K, r, \sigma)$" currently refers to "the price of a call".

Page 155

- In the first paragraph, "$S_T^* = S_t/B_t$" should be replaced by "$S* = S/B$" for consistency with the definition on page 120 in the first paragraph of Section 3.1.4. Note that in any case, the time indices don't match.

- Replace "Thus, the process W_t^*, ..." by "Thus, the process W^*, ...".

Page 156

- The last equation should read,

$$\begin{aligned} \tilde{N}(x) &= \int_{-\infty}^{x} N(u)du = uN(u)|_{-\infty}^{x} - \int_{-\infty}^{x} un(u)du \\ &= xN(x) + n(u)|_{-\infty}^{x} = xN(x) + n(x). \end{aligned}$$

I.e. the sign of the second term is flipped.

Page 157

- In the first equation, the sign of the second term is flipped. Using the put-call parity and Equation (3.77), we get

$$\begin{aligned} P_t &= C_t - (S_t - K) \\ &= \sigma\sqrt{T-t}n\left(d\left(S_t, T-t\right)\right) + \left(S_t - K\right)\left(N\left(d\left(S_t, T-t\right)\right) - 1\right) \\ &= \sigma\sqrt{T-t}n\left(d\left(S_t, T-t\right)\right) - \left(S_t - K\right)N\left(-d\left(S_t, T-t\right)\right). \end{aligned}$$

- Similarly, the sign of the second term in the third equation ($C_0 = \ldots$) should be a plus instead of a minus.

Chapter 4 - Cross-Currency Market Models

Page 183

- In the paragraph under Equation (4.3), replace "Hence for any $\hat{\eta}$ satisfying..." by "Hence for any ζ satisfying...".

Page 193

- in the second paragraph, replace "...does not accounts..." by "...does not account...".

Chapter 11 - Models of Instantaneous Forward Rates

Page 448

- In the equation for ϕ^2, you denote $F_Z(t,T)$ by F_t. This is inconsistent with the notation in the preceding line. I.e. I suggest either writing

$$\phi_t^1 = \mathcal{N}\left(\tilde{d}_1\left(F_Z(t,T), t, T\right)\right), \qquad \phi_t^2 = -K\mathcal{N}\left(\tilde{d}_2\left(F_Z(t,T), t, T\right)\right)$$

or

$$\phi_t^1 = \mathcal{N}\left(\tilde{d}_1\left(F_r, t, T\right)\right), \qquad \phi_t^2 = -K\mathcal{N}\left(\tilde{d}_2\left(F_t, t, T\right)\right).$$

Page 465

- In the Musiela parametrization, Equation (11.96), the terms $\sigma(t,x)$ should be replaced by $\sigma(t,t+x)$ or alternatively, you could define $\tilde{\sigma}(t,x) = \sigma(t,t+x)$ and then write

$$dr(t,x) = \left(\frac{\partial}{\partial x} r(t,x) + \tilde{\sigma}(t,x) \cdot \int_0^x \tilde{\sigma}(t,u)du\right) dt + \sigma(t,x) \cdot dW_t^*.$$

  Also note that there is a dot-product symbol missing before the integral.

Chapter 12 - Models of LIBOR

Page 474

- In the definition of $E\left(T_j, T_j\right)$, the term "$L\left(T_j\right)$" should be replaced by "$L\left(T_j, T_j\right)$". Although the meaning of this expression is clear from the context, this notation has neither been introduced nor been used before or afterwards.

Page 475

- In Equations (12.8), (12.9) as well as the unnumbered equation before, "$b(s,\cdot)$" should be replaced by "$b(t,\cdot)$".

Page 476

- In Equation (12.11), there is a closing bracket missing in the integrand - replace "$\left|\gamma\left(u, T_j, T_{j+1}\right|^2$" by "$\left|\gamma\left(u, T_j, T_{j+1}\right)\right|^2$".

Page 479

- "$t \in [0,T]$" should be replaced (three times in total) by "$t \in [0,T_0]$".

- It has been shown on the previous page, that the price of the j-th caplet with unit notional value is given by

$$\mathbf{Cpl_t^j} = \tilde{\delta}_j B\left(t, T_{j-1}\right) \mathbb{E}_{\mathbb{P}^{T_{j-1}}}\left[\left.\left(\tilde{\delta}_j^{-1} - B\left(T_{j-1}, T_j\right)\right)^+ \right| \mathscr{F}_t\right]$$

Thus, the j-th caplet is equivalent to a put option in the zero-coupon bond $B\left(\cdot, T_j\right)$ with option maturity in T_{j-1}, a strike price of $\tilde{\delta}_j^{-1}$ and an option notional value of $\tilde{\delta}_j$. By Proposition 11.3.1, the arbitrage price of this option is given by

$$\begin{aligned}
\mathbf{Cpl}_t^j &= \tilde{\delta}_j \left(\tilde{\delta}_j^{-1} B\left(t, T_{j-1}\right) \mathcal{N}\left(-h_2(t,T)\right) - B\left(t, T_j\right) \mathcal{N}\left(-h_1(t,T)\right)\right) \\
&= B\left(t, T_j\right) \left(\mathcal{N}\left(-h_2(t,T)\right) - \tilde{\delta} j F_B\left(t, T_j, T_{j-1}\right) \mathcal{N}\left(-h_1(t,T)\right)\right),
\end{aligned}$$

where

$$h_{1,2}(t,T) = \frac{\ln\left(\tilde{\delta}_j B\left(t, T_j\right) / B\left(t, T_{j-1}\right)\right) \pm \frac{1}{2} v^2(t,T)}{v(t,T)}$$

and

$$v^2(t,T) = \int_t^T ||b\left(u, T_j\right) - b\left(u, T_{j-1}\right)||^2 du.$$

We thus have

$$\begin{aligned}
-h_{1,2}(t,T) &= \frac{\ln\left(B\left(t, T_{j-1}\right) / B\left(t, T_j\right)\right) - \ln \tilde{\delta}_j \mp \frac{1}{2} v^2(t,T)}{v(t,T)} \\
&= \frac{\ln F_B\left(t, T_{j-1}, T_j\right) - \ln \tilde{\delta}_j \mp \frac{1}{2} v^2(t,T)}{v(t,T)}.
\end{aligned}$$

Obviously, $e_{1,2}(t,T)$ were supposed to be chosen such that $-h_{1,2}(t,T) = e_{2,1}(t,T)$. Although correct, I find this notation confusing since it is exactly the opposite of the one normally used. Furthermore, the definition of $v^2(t,T)$ in Lemma 12.3.1 should be

$$v^2(t,T) = \int_t^T ||\gamma(u, T+\delta, T)||^2 du.$$

Page 484

- In the last equation "$-\frac{1}{2}\sigma^2 t^2$" should be replaced by "$-\frac{1}{2}\sigma^2 t$".

Page 485

- In the equation for the caplet (second from the top), "$\kappa N\left(\hat{e}_1(t,T)\right)$" should be replaced by "$\kappa N\left(\hat{e}_2(t,T)\right)$".

Page 486

- Below the first equation, it says "...with a deterministic volatility function $\lambda(t, T+\delta)$...". Is there any reason to write "$\lambda(t, T+\delta)$" instead of just "$\lambda(t,T)$"?

Page 488

- In Equation (12.29), the upper limit of summation should be "$[\delta^{-1}(T-t)]$" instead of "$[\delta^{-1}T]$" as this defined $b(t,T)$ and not $b(0,T)$.

Page 489

- In the first equation, the drift coefficient should be "$L(t,T)\sigma^*(t,T+\delta)\cdot\lambda(t,T)$" instead of "$L(t,T)\sigma^*(t,T)\cdot\lambda(t,T)$" since by the following equation, we have

$$dL(t,T) = L(t,T)\lambda(t,T)\cdot dW_t^{T+\delta}.$$

I.e. $L(t,T)$ is a $\mathbb{P}^{T+\delta}$-martingale. The process $W^{T+\delta}$ is defined by

$$W_t^{T+\delta} = W_t^* - \int_0^t b(u, T+\delta)du.$$

Thus,

$$\begin{aligned} dL(t,T) &= L(t,T)\lambda(t,T)\cdot(W_t^* + \sigma^*(t,T+\delta)dt) \\ &= L(t,T)\sigma^*(t,T+\delta)\cdot\lambda(t,T)dt + L(t,T)\lambda(t,T)\cdot dW_t^*. \end{aligned}$$

Page 490

- In the middle of the page, "...and a family W^{T_j}, $j = 0,\ldots,n-1$ of processes..." should be replaced by "...and a family W^{T_j}, $j = 1,\ldots,n$ of processes..." since $L(t,T_0)$ is a $\mathbb{P}^{T_1}$-martingale, $L(t,T_1)$ is a $\mathbb{P}^{T_2}$-martingale and so on.

Page 492

- In the equation for $U_{m+1}(t,T_k^*)$, replace "δ_{n-m}" by "δ_{n-m+1}". By the definition at the top of page 490, we have

$$U_{n-j+1}(t,T_k) = \frac{B(t,T_k)}{B(t,T_j)} \quad \Rightarrow \quad U_j(t,T_k) = \frac{B(t,T_k)}{B\left(t,T_{j-1}^*\right)}.$$

Furthermore,

$$1+\delta_{n-m}L\left(t,T_{m+1}^*\right) = \frac{B\left(t,T_{m+1}^*\right)}{B(t,T_m^*)} \quad \Rightarrow \quad 1+\delta_{n-m+1}L(t,T_m^*) = \frac{B(t,T_m^*)}{B\left(t,T_{m-1}^*\right)}.$$

Thus,

$$\begin{aligned} U_{m+1}(t,T_k^*) &= \frac{B(t,T_k^*)}{B(t,T_m^*)} \\ &= \frac{B(t,T_k^*)}{(1+\delta_{n-m+1}L(t,T_m^*))\,B\left(t,T_{m-1}^*\right)} \\ &= \frac{U_m(t,T_k^*)}{1+\delta_{n-m+1}L(t,T_m^*)}. \end{aligned}$$

- For the same reason, the equation below should read

$$W_t^{T_m^*} = W_t^{T_{m-1}^*} - \int_0^t \frac{\delta_{n-m+1}L(u,T_m^*)}{1+\delta_{n-m+1}L(u,T_m^*)}\lambda(u,T_m^*)\,du.$$

Page 495

- The last equation should read

$$\frac{B(t,T_{k+1})}{G_t} = \prod_{j=1}^{m(t)}\left(1+\delta_j L(T_{j-1},T_{j-1})\right)^{-1} \prod_{j=m(t)+1}^{k+1}\left(1+\delta_j L(t,T_{j-1})\right)^{-1}.$$

I.e. the upper limit of the second product should be "$k+1$" instead of "k". This can be seen by computing

$$\begin{aligned}
\frac{B\left(t,T_{k+1}\right)}{B\left(t,T_{m(t)}\right)} &= \left(\frac{B\left(t,T_{m(t)}\right)}{B\left(t,T_{m(t)+1}\right)}\cdot\frac{B\left(t,T_{m(t)+1}\right)}{B\left(t,T_{m(t)+2}\right)}\cdot\ldots\cdot\frac{B\left(t,T_{k}\right)}{B\left(t,T_{k+1}\right)}\right)^{-1} \\
&= \left(\prod_{j=m(t)}^{k}\frac{B\left(t,T_{j}\right)}{B\left(t,T_{j+1}\right)}\right)^{-1} \\
&= \left(\prod_{j=m(t)}^{k}\left(1+\delta_{j+1}L\left(t,T_{j}\right)\right)\right)^{-1} \\
&= \prod_{j=m(t)+1}^{k+1}\left(1+\delta_{j}L\left(t,T_{j-1}\right)\right)^{-1}
\end{aligned}$$

Page 496

- There is a dot missing in the formula before (12.40), i.e. replace "$\zeta\left(t,T_j\right)b\left(t,T_{j+1}\right)$" by "$\zeta\left(t,T_j\right)\cdot b\left(t,T_{j+1}\right)$".

Page 506

- See my comment for page 479. I think "$t\in[0,T]$" should be replaced by "$t\in[0,T_0]$" in Proposition 12.6.1.

Page 507

- In the equation for I_2, there is one opening bracket too much - replace "$\ln\left(L\left(t,T_{j-1}\right)\right.$" by "$\ln L\left(t,T_{j-1}\right)$".

- Also, in the same equation, the total variance is denoted by $v_j^2(t)$ although this notation has not been introduced and Proposition 12.6.1 denotes this term by $\tilde{v}_j^2(t)$ instead. So I suppose "$v_j^2(t)$" should be replace by "$\tilde{v}_j^2(t)$". The same applies to the equation for I_1 on the next page.

Appendix A - An Overview of Itô Stochastic Calculus

Page 637

- In the sentence following the first formula, shouldn't it read "...belongs to the class $\mathcal{L}^2_{\mathbb{P}}(W)$."?

Page 639

- In the Theorem A.12.1, the notation suddenly changes when stating the linear growth condition. $a(t,x)$ should be replaced by $\mu(t,x)$ and $b(t,x)$ by $\sigma(t,x)$.

- I've only seen the Lipschitz continuity and linear growth conditions with the norms not being squared as in Theorem A.12.1. See for example Theorem 5.2.1 in Øksendal (2005) "Stochastic Differential Equations".